General theorems concerned with real roots of algebraic equations.

1) <u>Theorem</u>: Descarte's Rule of signs.

(a) the no. of positive roots cannot exceed the no. of changes of signs from positive to negative and from negative to ~~positive~~ positive in $f_n(x)$

(b) the no. of negative roots cannot exceed the no. of variations in $f_n(-x)$.

2) <u>Theorem</u>: If two real quantities 'a' and 'b' be substituted for 'x' in any polynomial $f_n(x)$ and if $f_n(a)$ and $f_n(b)$ are of opposite signs, then at least one or an odd no. of real roots of the equation, $f_n(x)=0$ lie b/w 'a' and 'b'. And if $f_n(a)$ and $f_n(b)$ are the same signs, then either no. of real root or an even no. of roots of $f_n(x)=0$ lie in b/w 'a' & 'b'.

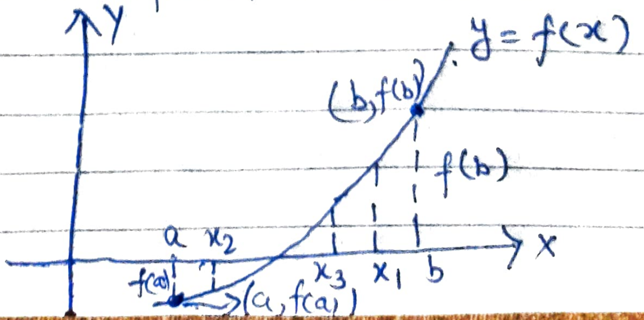
3) <u>Theorem</u>: Every equation of an odd degree has at least on real root opposite to that of (a_n/a_0), whereas every eqn of an even degree whose last term is negative has atleast two real roots, one positive and the other negative.

4) **Theorem**: The largest root of an equation $a_1x^n + a_2x^{n-1} + \dots + a_{n-1}x + a_n = 0$, may be obtained approximately by the root of the equation $a_0x + a_1 = 0$, or by the root of the quadratic equation $a_0x^2 + a_1x + a_2 = 0$ which is larger than the absolute value, similarly the smallest root can be obtained approximately by the root of the eqn $a_{n-1}x + a_n = 0$ or by the root of the eqn $a_{n-2}x^2 + a_{n-1}x + a_n = 0$ which is smaller than the absolute value.

Bisection Method:

This method is used for finding an approximate solution of the equation $f(x) = 0$ to the desired degree of accuracy and is based on the theorem which states that "the function $f(x)$ is continuous in the interval $[a, b]$ and $f(a), f(b)$ both have opposite signs, then the equation $f(x) = 0$ has atleast one ~~root~~ real root between $x = a$ and $x = b$".

Without loss of any generality, let $f(a)$ be negative and $f(b)$ be positive. Then atleast one of the equation $f(x) = 0$ lies in the open interval (a, b).



Let the first approximation to the root be obtained by bisecting the interval (a, b). Let it be x_1 which is given by $x_1 = \frac{a+b}{2}$

If $f(x_1) = 0$, then x_1 is the required root for $f(x) = 0$. Otherwise the root will lie either in the interval (a, x_1) or (x_1, b). according as $f(x_1)$ be +ve or -ve. Then we bisect the interval as before & continue the ~~interval~~ process until the root of eqn $f(x) = 0$ is found to a desired degree of accuracy.

In the above diagram. $f(x_1)$ is positive, so that the root lies b/w 'a' & 'x_1'. Then we bisect the interval (a, x_1) and get the second approximation to the root which is given by $x_2 = \frac{a+x_1}{2}$

If $f(x_2)$ is negative then the root will lie b/w x_1 & x_2. So to get the third approximation we again bisect (x_1, x_2) to get x_3 i.e., $x_3 = \frac{x_1+x_2}{2}$

We continue this process until we obtain the root of desired degree of accuracy.